CHAPTER 9 STRATEGIC REAL OPTIONS

So far all of the real options considered have been proprietary, in a monopoly context. No one can take away the investment opportunity, and the payoffs are dependent only on an exogenous price, not directly on the individual actions of other producers. Now we consider "strategic" options, typically duopolies, where there is the possibility of a leader with a first mover advantage (FMA), and then a follower.

FMA might be created by (1) the technological advantages of being first; (2) by patents if innovative and first; (3) by the brand image of a first mover, and (4) by organization and location advantage. However, being first may not be the same as being the best, since it may pay to wait and learn from the first mover's mistakes. One of the challenges is to quantify the first mover's advantages, and disadvantages. Frequent and public monitoring is sometimes feasible, for instance in the cases of public marketing of innovations, drug approval applications and patent applications. However, brand loyalty and differential pricing for the first mover is not always transparent, or measurable. For instance, for Internet service providers, only some companies provide frequent information on new accounts, churn and usage, and seldom provide detailed frequent information on sector profits (volume, unit revenues, unit costs).

There are many examples in R&D of the advantages of being first. The first company in an industry to have a website might buy cheaper domain names and obtain lower staff costs and better access to resources. In orphan drug status, the FDA may award the first mover a pre-emptive advantage, even prior to the completion of clinical trials. In general, once a patent is obtained as a result of R&D, the first mover may have an advantage for the number of years the patent is valid.

9.1 SIMPLE STRATEGIC REAL OPTIONS

Here is a simple strategic real option model where being first (the leader) results in a permanent market share advantage of α= (½, 1], see Tsekrekos (2003). V is the total market value, σ the value volatility, r the riskless rate, δ is the asset value yield, and μ the drift rate. Both the leader and follower invest K to obtain the market (value, V) or share thereof. The leader gets V perpetually until and if the follower enters, when then the leader's perpetual profit is reduced to αV thereafter, the follower obtaining (1-α)V. This approach can be simplified to an easy to use duopoly real options model. Where only the value V is stochastic, and α>.5 is the leader's market share, the positive root of the option elasticity measure is equation (9.0), where r-δ=μ.

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \qquad (9.0)$$

The follower's value function is:

$$\begin{cases} [(1-\alpha)V_F - K]\left(\dfrac{V}{V_F}\right)^{\beta_1} & V < V_F \\ (1-\alpha)V - K & V \geq V_F \end{cases} \qquad (9.1)$$

The follower's trigger is:

$$V_F^* = \frac{K\beta_1}{\beta_1 - 1}\frac{1}{1-\alpha} \qquad (9.2)$$

The leader's value function is:

$$\begin{cases} V - V^{\beta_1} * V_F^{(1-\beta_1)} + \alpha V_F\left(\dfrac{V}{V_F}\right)^{\beta_1} - K & V < V_F \\ \alpha V & V \geq V_F \end{cases} \qquad (9.3)$$

When $V<V_F$, the first line in equation 9.1 represents the follower's option to invest K in order to obtain (1-α)V. The second line represents the follower's value after investment. Note that the second row in the leader's value function assumes that the leader has invested first, so the cost of investment is not deducted when the follower invests. Usually the leader's trigger is established by equating the value functions of the leader and the follower, substituting the leader's trigger for

V in both equations (9.1) and (9.3) when $V<V_F$. In the special case where β_1=2, solve a quadratic for a closed-form solution.

The leader's trigger if β_1 =2 is:

$$V_L = \frac{1-\sqrt{1-4dK}}{2d} \qquad (9.4)$$

$$d = [V_F^{-\beta_1}][(1-\alpha)V_F - K] + (1-\alpha)V_F^{1-\beta_1}$$

Figure 9.1

	A	B	C
1	**Duopoly Competition with Permanent First-Mover Advantage**		
2	**Stochastic Value**		
3			
4	Risk-free Rate of Interest (r)	4.00%	
5	Yield	4.00%	
6	Volatility of Value (σ)	20.00%	
7	Leader's Market Share After Follower's Entry (α)	55.0%	
8	Total Market Value (V)	1,250,000	
9	Investment Cost (K)	1,000,000	
10	**Follower's Real Option Value**	**79,102**	IF(B8<B13,B16,B17)
11	**Leader's Real Option Value**	**91,797**	IF(B8<B13,B18,B19)
12			
13	**Investment Trigger Follower (V_F)**	**4,444,444**	(B21/(B21-1))*B9*(1/(1-B7))
14	**Investment Trigger Leader (V_L)**	1,229,636	B24
15			
16	Follower's Real Option Value (Before Entry)	79,102	(((1-B7)*B13-B9)*(B8/B13)^B21)
17	Follower's Real Option Value (After Entry)	-437,500	((1-B7)*B8-B9)
18	Leader's Real Option Value (Before Entry of F.)	91,797	B8-(B8^B21)*(B13^(1-B21))+B7*B13*(B8/B13)^B21-B9
19	Leader's Real Option Value (After Entry of F.)	687,500	(B7*B8)
20			
21	β_1	2.00	0.5-((B4-B5)/B6^2)+SQRT((((B4-B5)/B6^2)-0.5)^2+(2*B4/B6^2))
22	IF B21=2	QUADRATIC SOLUTION	
23	d	0.000000152	(B13^(-B21))*((1-B7)*B13-B9)+(1-B7)*B13^(1-B21)
24	B14=	1,229,636	(1-SQRT(1-4*B23*B9))/(2*B23)

Figure 9.1 is an example of a leader above threshold investment call option. The total market worth is $1,250,000, K=$1,000,000 for both leader and follower, and the leader's market share after the follower enters is 55%. The leader would hold the $1,250,000 market value by investing $1,000,000 until the follower enters, when then the leader would hold a value of .55*$1,250,000=$687,500. The interest rate is 4%, the asset value yield is 4%, and value volatility is 20%, so β_1=2. Figure 9.1 shows that the value of the real investment opportunity option for both the leader and the follower is small (total market value V is (1.25/2)=62.5% of the total investment cost). The leader should invest now, since $V_L < V$. The leader's primary value is based on monopoly profits until the follower enters, which should not be until V is 4.44/1.25=3.55 times the current market value.

9.2. STOCHASTIC PROFITS IN DUOPOLY GAMES

A similar approach is to consider before entry and post entry profits for both the follower and leader, where the total market may increase. Assume the leader after investment of a sunk cost K will have a profit flow of x (so that $V=x/(r-\mu)$), until a follower also makes the same sunk cost investment K, when then both firms have the same profit flow of x_2[1]. The real option value for the leader and the follower is established by finding the optimal stopping region, when it is optimal for those firms to invest.

Let V_1^F (x) denote the value of the follower in the stopping region, the region where it is optimal to invest and V_0^F (x) denote the value of the follower prior to investment. The optimal investment rule is found by solving for the boundary between the continuation and the stopping regions. The boundary is the trigger point x_F. If the value of the state variable is smaller than the trigger, the optimal decision for the follower is not to invest, i.e. to continue in the continuation region. If it exceeds the trigger, then the follower should invest. At the trigger point, two conditions must be satisfied: (1) the value matching condition makes explicit that when the state variable reaches the trigger the follower will invest so that $V_0^F(x_F) = V_1^F(x_F) - K$ and (2) the smooth-pasting condition requires that the first derivatives of the functions match at the boundary, $V_0^F{}'(x_F) = V_1^F{}'(x_F)$. These conditions imply:

$$x_F = \frac{K\beta_1(r-\mu)}{(\beta_1 - 1)} \qquad (9.5)$$

where β_1 is from equation (9.0). Thus the value function of the follower is:

$$V^F(x) = \begin{cases} \left(\frac{x_F}{r-\mu} - K\right)\left(\frac{x}{x_F}\right)^{\beta_1} & if \quad x < x_F \\ \frac{x_2}{r-\mu} - K & if \quad x \geq x_F \end{cases} \qquad (9.6)$$

[1] Each firm produces a unit output at zero variable cost, and the price follows a demand function P=Y D(Q), where Q=0,1,2. So $x=x_1$ is the profit function for Y D(1), and x_2 for Y D(2). The condition $x>x_2$ implies that the leader's monopoly profits exceed the equal profits for the leader and the follower after the follower enters. The FMA is only the leader's monopoly profits until the follower enters.

Until the follower enters the market, there exists an optimal time for the leader to enter that will maximize the firm's value. The leader's trigger is the solution to equating the first line of 9.7 to the first line of 9.6 substituting x_L for x. Until that moment the firm should wait to invest and its value is explained by the option to wait. Having entered the market, the leader will enjoy monopolistic profits until the moment that the follower enters the market, and will share profits with the follower afterwards. The general solution for the leader is:

$$V^L(x) = \begin{cases} \frac{x_L}{r-\mu}[1-\left(\frac{x_L}{x_F}\right)^{\beta_1 - 1}] + \frac{x_2}{r-\mu}\left(\frac{x}{x_F}\right)^{\beta_1} - K & if \quad x < x_F \\ \frac{x_2}{r-\mu} & if \quad x \geq x_F \end{cases} \qquad (9.7)$$

Figure 9.2

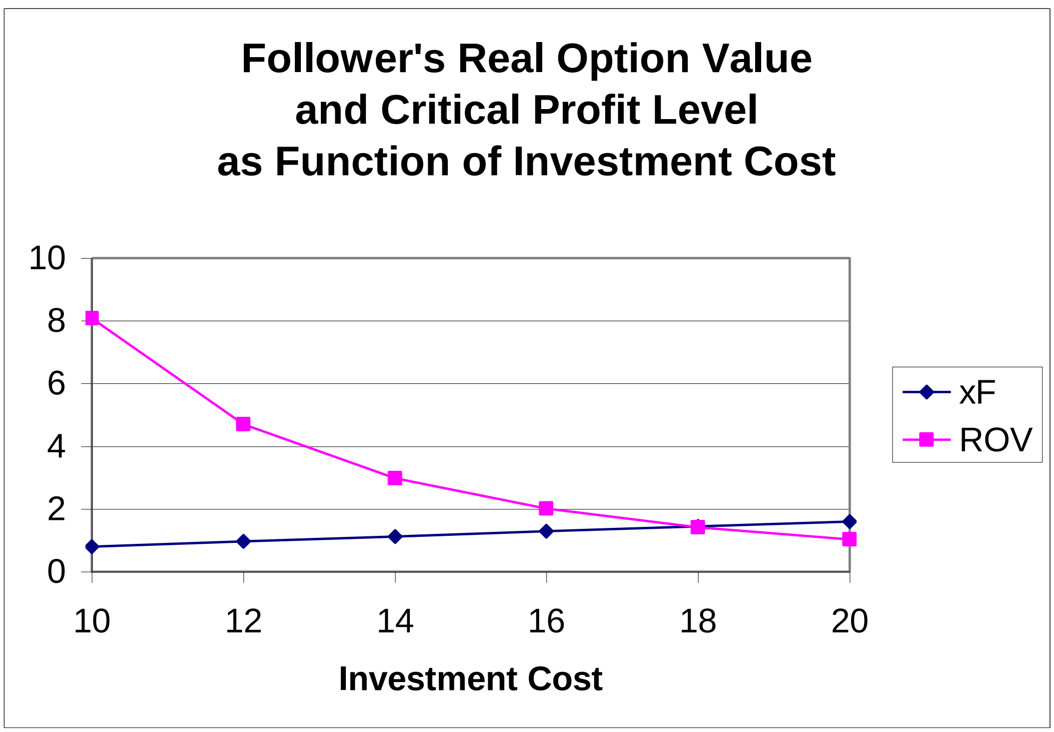
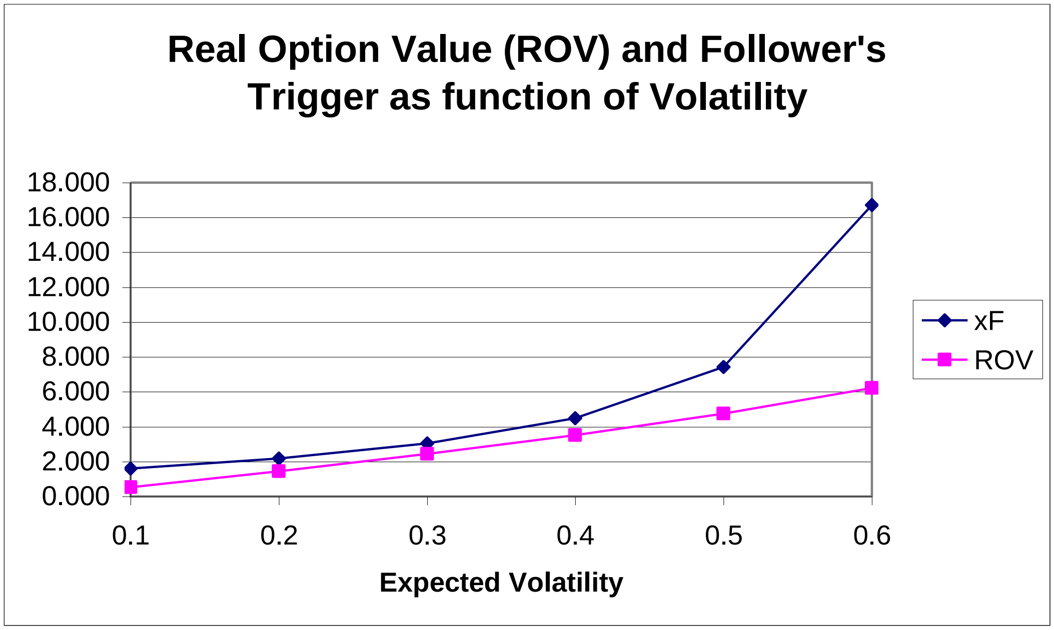


Figure 9.2 shows that the follower's critical profitability (x_F) is higher, and the real option value (ROV) is lower, the higher the investment cost K.

Figure 9.3 shows that both the follower's critical profitability and real option value increase with profit volatility.

Figure 9.3



9.3 RIVALRY UNDER PRICE and QUANTITY UNCERTAINTY

Consider a market with high volume products such as raw materials, food or fuel, so that the number of units sold is practically a continuous variable. Suppose that both the profit per unit and the number of units follow different but possibly correlated geometric Brownian motion processes, see Paxson and Pinto (2005). Let P represent the profit per unit sold and Q the quantity sold in a market by a follower. Assume that each variable follows a geometric Brownian motion of the form:

$$dP = \mu P\,dt + \sigma P dz_1 \tag{9.8}$$

$$dQ = \omega Q\,dt + \alpha Q\,dz_2 \tag{9.9}$$

where μ and ω are the expected multiplicative trends of P and Q, σ and α are the volatilities, and dz_1 and dz_2 the increments of a Wiener process. The two variables may be correlated with correlation coefficient ρ. Consider two firms with the same investment cost, K, that are contemplating entering a new market. The firm that enters first (defined as the leader) will acquire a FMA, that is the leader is assumed to have a higher share of the market after the follower enters. The following differential equation for an idle follower is:

$$\frac{1}{2}\frac{\partial^2 P_0^F}{\partial P^2}\sigma^2 P^2 + \frac{1}{2}\frac{\partial^2 P_0^F}{\partial Q^2}\alpha^2 Q^2 + \frac{\partial^2 P_0^F}{\partial P \partial Q} PQ\rho\sigma\alpha + \mu P\frac{\partial P_0^F}{\partial P} + \omega Q\frac{\partial P_0^F}{\partial Q} - rP_0^F = 0 \tag{9.10}$$

Equation (9.10) explains the movements in the value function of an idle follower and is subject to the usual boundary conditions. The first boundary condition is the value matching that gives the value of $P_0^F(P,Q)$ at which the follower should invest. The second boundary condition is the smooth pasting that assures that the derivatives of the two functions (before and after the follower enters the market) are equal at the investment point.

Let $X = PQ$ denote the total profit for the follower, implying that $P_0^F(X) = P_0^F(P,Q)$. After the appropriate substitutions, equation (9.10) can be written as:

$$\frac{1}{2}X^2\frac{d^2 P_0^F(X)}{dX^2}\left[\sigma^2 + \alpha^2 + 2\rho\sigma\alpha\right] + X\frac{dP_0^F(X)}{dX}\left[\rho\sigma\alpha + \mu + \omega\right] - rP_0^F(X) = 0 \tag{9.11}$$

Equation (9.11) involves the following characteristic quadratic function:

$$\frac{1}{2}(\sigma^2 + \alpha^2 + 2\rho\sigma\alpha)\beta(\beta - 1) + (\rho\sigma\alpha + \mu + \omega)\beta - r = 0 \tag{9.12}$$

The positive root of Equation (9.12) is:

$$\beta_1 = \frac{1}{2} - \frac{\mu - \omega}{z^2} + \sqrt{\left(\frac{\mu - \omega}{z^2} - \frac{1}{2}\right)^2 + \frac{2r}{z^2}} \tag{9.13}$$

where $z^2 = \alpha^2 + \sigma^2 + 2\rho\sigma\alpha$.

The solution of equation (9.11) is:

$$P_0^F(X) = AX^{\beta_1} + BX^{\beta_2} \tag{9.14}$$

We know that as X increases, the value function of the follower has to increase and that equation (9.14) has to be finite, thus B equals zero. Equation (9.14) is subject to the value-matching condition:

$$P_0^F(X_F) = \frac{X_F}{r-\mu-\omega} - K \tag{9.15}$$

where X_F is the follower's trigger value, i.e. the value of X at which the follower should enter the market, and is also subject to the smooth-pasting condition:

$$\frac{dP_0^F(X_F)}{dX_F} = \frac{1}{r-\mu-\omega} \tag{9.16}$$

Equations (9.14), (9.15) and (9.16) imply that:

$$X_F = \frac{K(r-\mu-\omega)\beta_1}{\beta_1 - 1} \tag{9.17}$$

Thus the value function of the follower, $P(F)$, is given by:

$$P(F) = \begin{cases} \dfrac{K}{\beta_1 - 1}\left(\dfrac{X}{X_F}\right)^{\beta_1} & X < X_F \\ \dfrac{X}{r-\mu-\omega} - K & X \geq X_F \end{cases} \tag{9.18}$$

Equation (9.18) describes the value function of the follower before and after the trigger is hit. Before the trigger X_F is hit, the follower has not yet entered the market and its value function is a monopolist perpetual American option to invest second in a new market. At the trigger, the follower invests and after that its value function is the perpetuity. Let m* be an absolute value larger than one that when multiplied by Q results in the number of units sold by the leader while alone in the market. Let m be a value larger than one, but smaller than m*, that when multiplied by Q results in the number of units sold by the leader after the follower enters the market. $Q(m-1)$ represents the first-mover advantage, in terms of number of units, after the follower enters the market.[2] The value function of the leader, *P(L)*, is given by:

$$P(L)=\begin{cases}\left(\dfrac{X}{X_F}\right)^{\beta_1}\dfrac{K\beta_1}{\beta_1-1}(m-m^*)+\dfrac{Xm^*}{r-\mu-\omega}-K & X<X_F\\ \dfrac{Xm}{r-\mu-\omega} & X\geq X_F\end{cases}\tag{9.19}$$

Figure 9.4

	A	B	C
1	**RIVALRY UNDER PRICE & QUANTITY UNCERTAINTY**		
2			
3	r	0.04	
4	u	0.00	
5	w	0.00	
6	σ	0.20	
7	σ^2	0.04	
8	α	0.20	
9	α^2	0.04	
10	ρ	-0.50	
11	z^2	0.04	B7+B9+2*B6*B8*B10
12	β	2.00	0.5-(B4-B5)/(B11)+SQRT(((B4-B5)/(B11)-0.5)^2 + 2*B3/(B11))
13	P	0.075	
14	Q	10.00	
15	X	0.750	
16	K	12.5	
17	X_F	1.000	B12*(B3-B5-B4)*B16/(B12-1)
18	Option	7.031	B16/(B12-1)*((B15/B17)^B12)
19	NPV	6.250	B15/(B3-B4-B5)-B16
20	Follower ROV	7.031	IF(B15<B17,B18,B19)
21	m	1	
22	m*	2	
23	Leader before	10.938	(B15/B17)^B12*B16*B12/(B12-1)*(B21-B22)+B15*B22/(B3-B5-B4)-B16
24	Leader after	18.750	B15*B21/(B3-B5-B4)
25	Leader ROV	10.938	IF(B15<B17,B23,B24)
26	X_L	**0.333**	
27	Solver	0.0000	Set B27=0, Changing B26.
28	Quadratic	**0.333**	
29	Quadratic	((-B22/(B3-B4-B5))+SQRT(4*(B16^2)*(2*(B21-B22)-1)+(-B22/(B3-B4-B5))^2))/(2*B16*(2*(B21-B22)-1))	
30	Solver	((B26/B17)^B12)*B16*B12/(B12-1)*(B21-B22)+B26*B22/(B3-B4-B5)-B16-B16/(B12-1)*(B26/B17)^B12	

Numerically the leader's trigger, X_L, can be found by solving the following non-linear equation (where X_L is the unknown):

$$\left(\frac{X_L}{X_F}\right)^{\beta_1}\frac{K\beta_1}{\beta_1-1}(m-m^*)+\frac{X_L m^*}{r-\mu-\omega}-K-\frac{K}{\beta_1-1}\left(\frac{X_L}{X_F}\right)^{\beta_1}=0\tag{9.20}$$

[2] Suppose, for example, that while alone in the market, the leader sells 15 units of a certain product and that after the follower enters the leader sells 10 units and the follower sells 5 units. In this case *Q=5*, *m*=3*, and $m=2$.

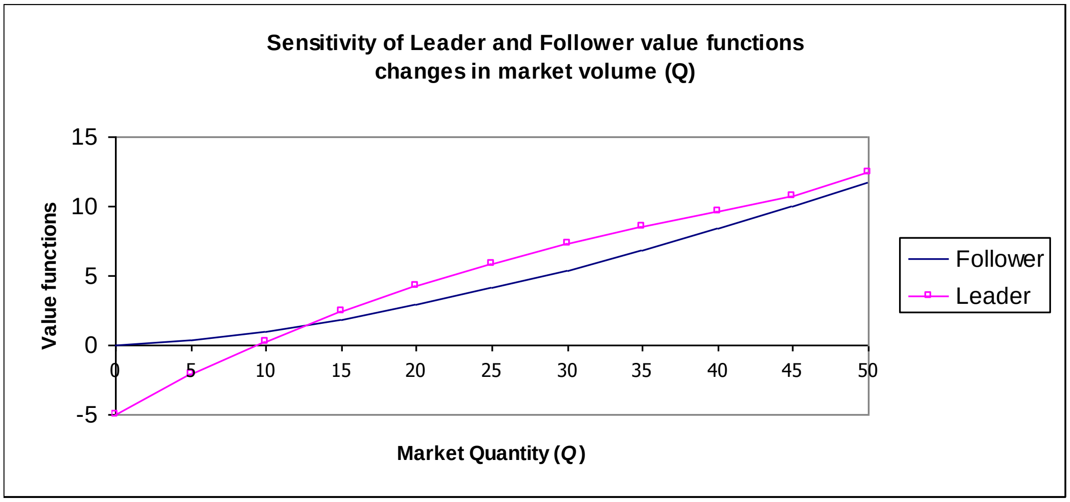
Figure 9.4 shows that when β_1=2, and the leader has a 0% market share advantage, the P*Q=.75, K=12.5, the leader's trigger of .33 indicates that the leader should invest now, but the follower should delay investing until X is 133% more than the current level. Nevertheless, the follower has significant real option entry value. Note that the leader in this illustration does not have any permanent market share advantage. The value of the leader and the follower is the same after the follower entry, so the primary value for the leader is the monopoly market share before the follower enters.

Note also that there is a closed form solution for the leader's trigger, when β_1=2, and when the follower's trigger =1, based on the quadratic equation solution.

$$X_L = \frac{\frac{-m^*}{r-\mu-\omega} + \sqrt{-4 * K^2[2(m-m^*-1] + (\frac{-m^*}{r-\mu-\omega})^2}}{2K*[2(m-m^*)-1]} \tag{9.21}$$

The value function of the leader is almost always larger than that of the follower, except when the number of units is very low. At this point the follower has not yet entered the market, and if the leader were already in the market it would have been better off being a follower. When the follower enters the market the two functions get closer. The value function of the leader is more complicated than the one of the follower. It is concave until the moment the follower enters, and at that moment its slope becomes discontinuous. This happens because although the number of units is increasing, they are also approaching the trigger of the follower, i.e. the negative effect of the entry of the follower increases. It can also be seen in Figure 9.5 that there is a point where the two functions meet. Before this point a firm would be better off being a follower, and after that a leader.

Figure 9.5



9.4 RIVALRY UNDER REVENUE and INVESTMENT COST UNCERTAINTY

Let R stand for total annualized profits ("termed" return) and K for investment costs, with both variables following a geometric Brownian motion:

$$dR = \mu Rdt + \sigma Rdz \tag{9.22}$$

and

$$dK = \omega Kdt + \alpha Kdz \tag{9.23}$$

where μ and ω are the expected gain of R and K respectively or, in other words the drift of the Brownian motion; σ and α are the volatilities and ρ the correlation coefficient, see Paxson and Melmane (2009). As before, two companies consider entering a new market, with the condition that the first mover, the leader, has an incentive to pre-emption, thereby obtaining a competitive advantage m (dominant market share).

The differential equation of the value function of an idle follower is:

$$\frac{1}{2}\frac{\partial^2 P_0^F}{\partial R^2}\sigma^2 R^2 + \frac{1}{2}\frac{\partial^2 P_0^F}{\partial K^2}\alpha^2 K^2 + \frac{\partial P_0^F}{\partial R \partial K} RK\sigma\alpha\rho + \mu R\frac{\partial P_0^F}{\partial R} + \omega K\frac{\partial P_0^F}{\partial K} - rP_0^F = 0 \tag{9.24}$$

This equation should be subject to the usual two boundary conditions; the first one is the value matching. Note that at the point where the value of the option equals the present value of the total return minus the investment, the investment is made and consequently all the uncertainty related to the investment cost disappears.

Let now the substitution be $Y = \frac{R}{K}$ implying that $P_0^F(R,K) = Kp\left(\frac{R}{K}\right) = Kp(Y)$.

After all the substitutions, (9.24) can be written as:

$$\frac{1}{2}Y^2 p''(Y)\gamma^2 + Yp'(Y)[\omega - \mu] - \omega p(Y) = 0 \tag{9.25}$$

where: $\gamma^2 = \sigma^2 + \alpha^2 - 2\sigma\alpha\rho$ (9.26)

The characteristic quadratic function of equation (9.25) is:

$$\frac{1}{2}\gamma^2 \beta(\beta - 1) + (\mu - \omega)\beta - (r - \omega) = 0 \tag{9.27}$$

with one solution the positive root:

$$\beta_1 = \frac{1}{2} - \frac{(\mu - \omega)}{\gamma^2} + \sqrt{\left(\frac{\mu - \omega}{\gamma^2} - \frac{1}{2}\right)^2 + \frac{2(r - \omega)}{\gamma^2}} \tag{9.28}$$

The follower's trigger is:

$$Y_F = \frac{(r - \mu)\beta_1}{\beta_1 - 1} \tag{9.29}$$

and the value function of the follower is:

$$V(F) = \begin{cases} \dfrac{K}{\beta_1 - 1}\left(\dfrac{Y}{Y_F}\right)^{\beta_1} & Y < Y_F \\ \dfrac{R}{r - \mu} - K & Y \geq Y_F \end{cases} \tag{9.30}$$

It is assumed that the leader will always have an advantage over the follower. Before the latter enters the market, the leader receives Rm*, where m* is a factor that multiplied by the return gives the total return of a monopoly. After the follower enters the market, the leader will still have an advantage, m, which is a multiplicative factor, larger than one, that can represent either larger market share or higher prices. The value function of the leader is:

$$V(L)=\begin{cases}\dfrac{Rm*}{r-\mu}+\left(\dfrac{Y}{Y_F}\right)^{\beta_1}\dfrac{\beta_1 K}{\beta_1-1}(m-m*)-K & Y\le Y_F\\ \dfrac{Rm}{r-\mu} & Y>Y_F\end{cases} \tag{9.31}$$

The trigger for the leader must be obtained numerically, equating the first line of equation (9.31) to the first line of equation (9.30), substituting Y_L for Y and $K*Y_L$ for R. Figure 9.6 shows that it is convenient to convert V (the present value of future cash flows) into R ($R=(r-\mu)V$), especially in the case where cash flows are irregular. Under certain restrictive conditions by expressing the triggers multiplied by K so that the follower's trigger might equal 1, there is a closed-form solution for the leader's trigger, if $\beta_1=2$.

Figure 9.6

	A	B	C
1	**RIVALRY UNDER NET REVENUE & INVESTMENT COST UNCERTAINTY**		
2			
3	Riskless rate	**0.05**	
4	Value Drift	**0.03**	
5	Cost Drift	**0.03**	
6	Value Volatility	**0.500**	
7	Cost Volatility	**0.500**	
8	Correlation	**0.000**	
9	σ^2	0.250	
10	α^2	0.250	
11	γ^2	0.500	B9+B10-2*B6*B7*B8
12	β_1	1.108	0.5+(B5-B4)/B11+SQRT(((B5-B4)/B11-0.5)^2+2*B5/B11)
13	PV of CF (F)=V	**1219.632**	
14	Eq AnnualRevenue	24.393	B13*(B3-B4)
15	K	**1155.916**	
16	Y Current	**0.021**	B14/B15
17	Y_F Trigger	**0.205**	(B3-B4)*B12/(B12-1)
18	Follower Before	**860.463**	B15/(B12-1)*(B16/B17)^B12
19	Follower After	**63.716**	B14/(B3-B4)-B15
20	Follower Value	**860.463**	IF(B16<B17,B18,B19)
21	m (L's Market A/F)	**4.000**	
22	m*(Total Market)	**5.000**	
23	Leader Before	**3988.613**	B14*B22/(B3-B4)+(B16/B17)^B12*B12*B15/(B12-1)*(B21-B22)-B15
24	Leader After	**4878.528**	B14*B21/(B3-B4)
25	Leader Value	**3988.613**	IF(B16<B17,B23,B24)
26	Follower at Y_L	189.214	B15/(B12-1)*(B28/B17)^B12
27	Leader at Y_L	189.214	B15*B28*B22/(B3-B4)+(B28/B17)^B12*B12*B15/(B12-1)*(B21-B22)-B15
28	Y_L Trigger	**0.005**	SOLVER: SET B29=0, CHANGING B28.
29	Y_F-Y_L	0.000	B27-B26
30	Y_F Trigger *K	**236.631**	
31	USE: INPUTS INTO BOLD BLUE CELLS		
32	SOLVER: SET B29=0, CHANGING B28.		
33			
34	Note that the quadratic solution cannot be used in this case.		

Figure 9.6 illustrates an in-the-money investment opportunity for the leader. Both the leader and follower have the same investment cost, but the leader obtains the entire V before the follower enters, and (4/5)V afterwards. The trigger for the leader is below the current Y, so the leader should invest now; the trigger for the follower is above the current Y, so the follower should wait.

SUMMARY

In this chapter there are four models for evaluating investment opportunities and optimal investment timing in different semi-competitive environments. All of these models assume a duopoly, where the follower follows the basic American perpetuity or perpetual exchange rules, and the leader has an advantage either of permanent market share dominance, and/or pre-emptive timing (that is invests first and has monopoly market share until the follower enters). These models differ primarily in the expression of market share, or number of factors. The second model allows the total market to increase as the follower enters. The first, third and fourth models assume the total market is not affected by the entry of the follower, but that the leader has a reduced market share after the follower enters. The third and fourth models are multi-factor, with price and quantities stochastic in the third model, and net revenue and investment cost stochastic in the fourth model. An extension to four factors is not difficult.

There are several other models of pre-emption and hysteresis, see Azevedo and Paxson, 2014. Some models consider jump processes for new competitive entry. Other plausible models have a stochastic market share and/or profitability. Followers and leaders might learn from each other, which may affect either pricing, or market share, or production cost. Possible pre-emption models might consider adjusting real option value for convenience yield, or assume higher/lower implementation costs, as firms enter the market. In general, pre-emption will be less likely if cash flows naturally are mean reverting. Other applications of pre-emption models include where creditors may foreclose first, informed investors buy or sell first, and where some contingent claim holders exercise their options first.

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EXERCISE 9.1 Julia Ma is contemplating becoming a leader in a small Real Options class, where there is just one other student, Mark Song. This unusual class involves a possible lifetime of learning and earning, where students are allowed to concentrate their learning time and efforts. Julia believes her lifetime earnings after the RO class will be worth $1,000,000 "V" (if there is no competitive follower) but the investment cost (including her current time and effort) is worth $1,000,000. She believes Mark values his current time and effort the same. Each believe the annual volatility of earnings worth is 20%, their earnings yield 4%, and the riskless interest rate is 4%. Both Julia and Mark suppose that the follower (second in the class) will choose to put forth time and effort later than the leader, but will then achieve lifetime earnings worth only 50% of the leader's. When Mark enters the "real options market", Julia's worth will be reduced to two-thirds (α) of the previous worth. Should Julia become the leader, and at what level of her earnings worth (when alone) should Mark put forth his efforts? State your assumptions clearly and provide some illustrative numbers.

PROBLEM 9.2 As a result of taking this special Real Options class, Julia believes the leader will produce a unique RO package that will be priced at P and sell Q in perpetuity. It is likely that P=$.10, Q=5, K=12.5, β_1=2, the leader will sell twice the follower's Q until the follower enters, but the same after. When should Julia become the leader, and at what level of P*Q should Mark put forth his efforts?

PROBLEM 9.3 This infernal Real Options class enables the leader, Julia, to produce a unique RO package that will have an expected net revenue R of $.50 in perpetuity, the investment cost K is expected to be 12.5, but both are uncertain, β_1=2.. The leader will sell twice the follower until the follower enters. When should Julia become the leader, and at what level of R/K should Mark put forth his efforts?

PROBLEM 9.4 Julia Ma is contemplating becoming a leader in a small Real Options class, where there is just one other student, Mark Song. This unusual class involves a possible lifetime of learning and earning, where students are allowed to concentrate their learning time and efforts when they consider appropriate. Julia believes that the leader if alone in the market will achieve lifetime earnings worth $1,250,000 "V" (if there is no competitive follower) but the investment cost (including her current time and effort) is worth $1,000,000. She believes Mark values his current time and effort the same. Each believe the annual volatility of earnings worth is 40%, and the riskless interest rate is 5%. Both Julia and Mark suppose that the follower (second in the class) will choose to put forth time and effort later than the leader, but will then achieve lifetime earnings worth only 35% of the total market. When Mark enters the "real options market", Julia's worth will be reduced to 65% (α) of the previous worth. Should Julia become the leader, and at what level of her earnings worth (when alone) should Mark put forth his efforts?

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PROBLEM 9.5 As a result of taking this special RO class, Julia believes the leader will produce a unique RO package that will be priced at P and sell Q in perpetuity. It is likely that P=$.20, Q=5, K=20, the leader will sell four times the follower until the follower enters, and double thereafter. The volatility of P is 40%, Q 30%, the correlation is -.2, and r=4%. When should Julia become the leader, and at what level of P*Q should Mark put forth his efforts?

PROBLEM 9.6 This wonderful RO class enables the leader, Julia, to produce a unique RO package that will have an expected net revenue R of $40,000 in perpetuity, the investment cost K is expected to be $1,000,000, but both are uncertain. The leader will sell three times the follower until the follower enters, and double thereafter. The volatility of R is 60%, K 10%, the correlation is .2, r=4%, and the investment cost escalation is 2%. When should Julia become the leader, and at what level of R/K should Mark put forth his efforts?